

When is a subset a subspace

1. A 10-dimensional vector may represent a sequence of ten temperature readings in degrees Celsius. Technically, do all possible readings form a subspace of all possible temperatures? Consider absolute zero or 0 K.

2. For navigation, we often model the Earth as literally a flat Earth: road navigation seldom takes into account changes in elevation. Is it unreasonable to model the Earth locally as if it was $\mathbf{R}^2$? Would you model the Earth as a two-dimensional surface if you were determining the navigation of jets? For example, if you have a globe, try to determine which direction you would have to fly to get to Alexandria, Egypt, which is south of Waterloo, to follow the shortest path to that city.

3. Engineers will very often approximate complex functions using simpler functions. For example, a synthesizer uses oscillators to generate different frequencies. Thus, with four oscillators, you could generate the sound $\cos(2\pi 440 t)$ [middle A], $\cos(2\pi 550 t)$ [C#], $\cos(2\pi 660 t)$ [E], $\cos(2\pi 880 t)$ [A one octave higher]. Thus, all possible sounds would be all linear combinations of these functions:

$$\alpha_1 \cos(2\pi 440 t) + \alpha_2 \cos(2\pi 550 t) + \alpha_3 \cos(2\pi 660 t) + \alpha_4 \cos(2\pi 880 t)$$

Does this form a subspace of all possible sounds?

Note, I'm not providing the frequencies of a well-tempered klavier, but rather, the C# and E most appropriately tuned for a song played in the key of A. In a well-tempered klavier, C# and E would have frequencies $440\left(\sqrt[12]{2}\right)^4 \approx 554.37$ and $440\left(\sqrt[12]{2}\right)^7 \approx 659.26$, respectively. You will note that $440\left(\sqrt[12]{2}\right)^{12} = 880$.